UNIVERSITÉ DE TECHNOLOGIE DE BELFORT-MONTBÉLIARD

Kinematics

RO51 - Introduction to Mobile Robotics

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https://yzrobot.github.io/

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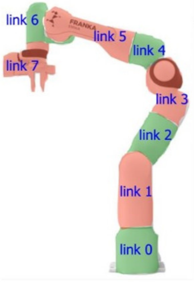
What is kinematics

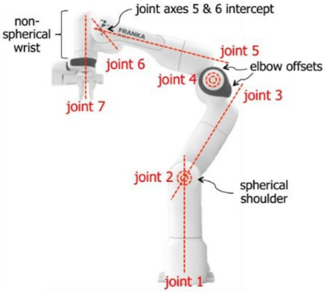
- A subfield of physics, developed in classical mechanics.
- Describes the motion of points, bodies (objects), and systems of bodies (groups of objects).
- Without considering the forces that cause them to move.

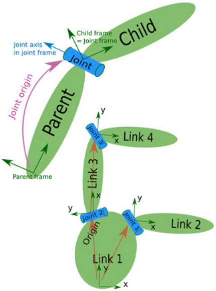
- A kinematics problem begins by:
  - describing the geometry of the system,
  - declaring the initial conditions of any known values of position, velocity and/or acceleration of points within the system.
- Then, the position, velocity and acceleration of any unknown parts of the system can be determined using arguments from geometry.
  - The study of how forces act on bodies falls within kinetics, not kinematics.

Kinematics in mobile robotics

- Essential for hardware and software design.
- Many problems are similar to industrial robotic arms.
  - Move from one pose to another in the workspace.







Kinematics in mobile robotics

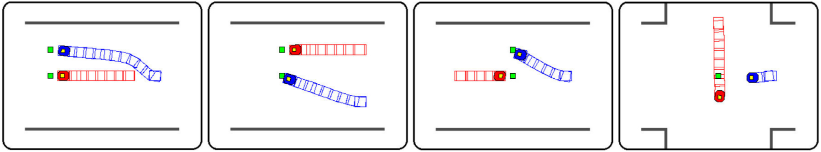
Mobile robots vs. Robotic arms

- Main difference: pose estimation
  - Robotic arms: one end fixed to the environment, the position of the arm's end effector can be easily and accurately measured.
  - Mobile robots: self-contained automaton, no direct way to measure a mobile robot's position instantaneously.

- Precisely measuring the pose of a mobile robot is a challenging task:
  - slippage
  - sensor errors
  - map and so forth

Kinematics in mobile robotics

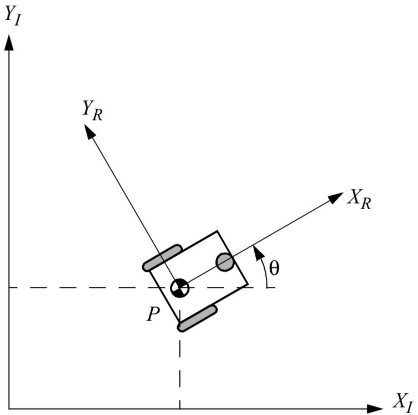
- Description of the robot motion varies according to the wheel type (c.f. locomotion).
  - Each wheel imposes constraints on the robot's motion.
- Forward kinematic models (vs. inverse kinematic models in robotic arms).
- Robot kinematics is the foundation of robot motion planning.



Wheeled robots

Modeling

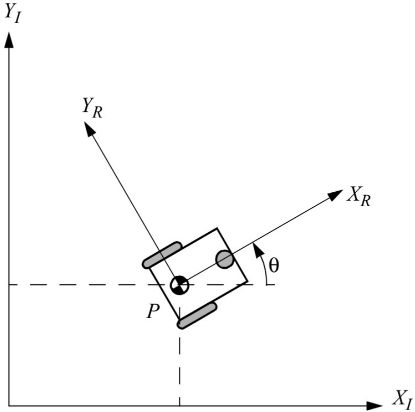
- A rigid body on wheels
- Operate on a horizontal plane
- *dimensionality* $= 3$
  - 2 for position in the plane
  - 1 for orientation along the vertical axis
- Global reference frame: $O : \{X_I, Y_I\}$
- Robot position: P (usually the center of rotation)
- Robot local reference frame: $\{X_R, Y_R\}$



Wheeled robots

Modeling

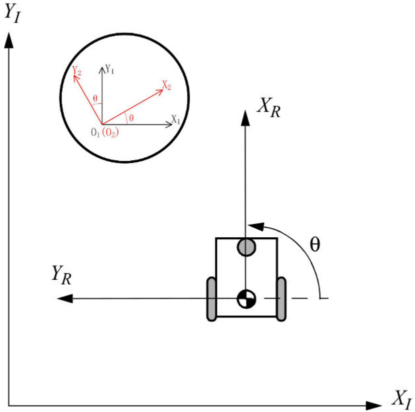
- Robot pose: $\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$
  - x, y: coordinates of P in the global reference system
  - θ: angular difference between the global and local reference frames
- How to describe the motion of the robot: $\dot{\xi}_R$?



Wheeled robots

Modeling

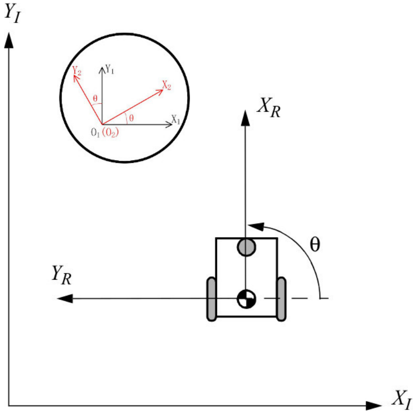
- $\dot{\xi}_R = R(\theta)\dot{\xi}_I$
- $R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $R(\theta)$ is used to map motion in the global reference frame to motion in the local reference frame (c.f. rotation matrix).
- $R(\theta)$ is an orthogonal rotation matrix.



Wheeled robots

Modeling

- If $\theta = 90^\circ$, then
  $$R(\tfrac{\pi}{2}) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
- Given some velocity $(\dot{x}, \dot{y}, \dot{\theta})$ in the global reference frame, then
  $$\dot{\xi}_R = R(\tfrac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$



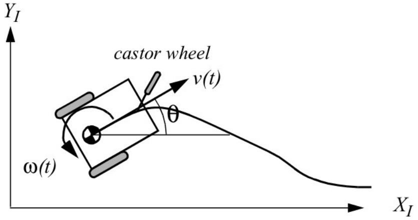
Wheeled robots

Forward kinematic model

- Differential drive robot:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$

  - l: distance from the drive wheel to P
  - r: diameter of the drive wheel
  - $\dot{\varphi}_1, \dot{\varphi}_2$: spinning speed of the drive wheels

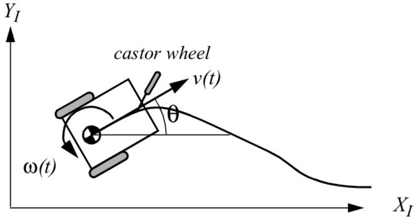


- $\dot{\xi}_I = R(\theta)^{-1}\dot{\xi}_R$ (inverse matrix)

Wheeled robots

Forward kinematic model

- Suppose that the robot moves forward along $+X_R$
- Translation velocity:
  - $\dot{x_{r1}} = \frac{r\dot{\varphi}_1}{2}$
  - $\dot{x_{r2}} = \frac{r\dot{\varphi}_2}{2}$
  - $\dot{y_R} = 0$ (sideways motion impossible)
- Rotation velocity:
  - $\omega_1 = \frac{r\dot{\varphi}_1}{2l}$
  - $\omega_2 = \frac{-r\dot{\varphi}_2}{2l}$



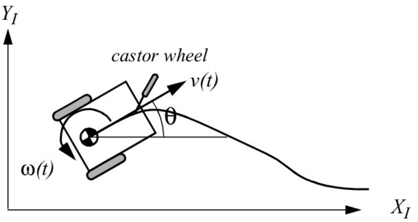
- $\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$

[1] If one wheel spins while the other wheel contributes nothing and is stationary, since P is halfway between the two wheels, it will move instantaneously with half the speed.

Wheeled robots

Forward kinematic model

- $R(\theta)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$



- Suppose that, $\theta = \frac{\pi}{2}$, $l = 1$, $r = 1$, $\dot{\varphi}_1 = 4$, and $\dot{\varphi}_2 = 2$, then

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

=> The robot will move instantaneously along the y-axis of the global reference frame with speed 3 while rotating with speed 1.

Wheeled robots

Wheel kinematic constraints

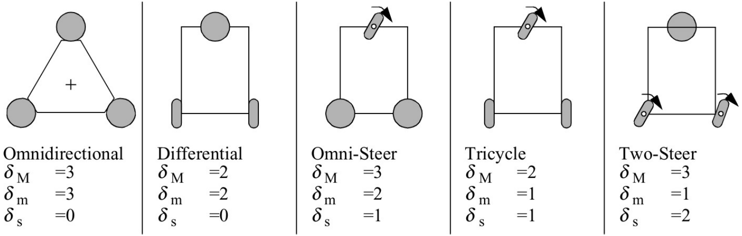
The establishment of kinematic models for different mobile robot chassis designs requires further description of the constraints of each wheel on the robot's motion.

- Fixed standard wheel
- Steered standard wheel
- Castor wheel
- Swedish wheel
- Spherical wheel

Wheeled robots

Maneuverability

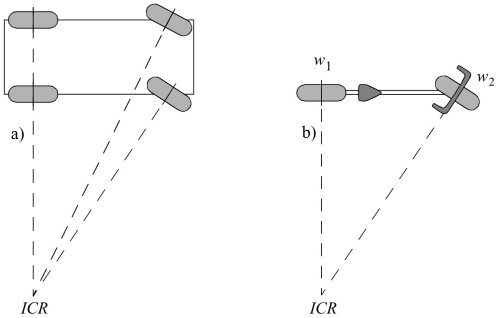
- The overall degrees of freedom that a robot can manipulate, called the degree of maneuverability δ_M , can be readily defined in terms of mobility and steerability: $\delta_M = \delta_m + \delta_s$
  - δ_m: ability of the robot to move directly in the environment (constrained by the wheels) => the DOFs that the robot manipulates directly
  - δ_s: independently controllable steering parameters => the DOFs that the robot manipulates indirectly



Wheeled robots

Maneuverability

- Two robots with the same δ_M are not necessarily equivalent.
- For any robot with $\delta_M = 2$ the ICR (Instantaneous Center of Rotation) is always constrained to lie on a line.
- For any robot with $\delta_M = 3$ the ICR can be set to any point on the plane.



Wheeled robots

Maneuverability

- maneuverability = control DOF $\neq$ DOF
  - Car (i.e. Ackerman vehicle):
    - $\delta_M = 2$: one for steering and the second for the drive wheels
    - $DOF = 3 : (x, y, \theta)$
- Differentiable DOF (DDOF):
  - Number of independently achievable velocities.
  - Always equal to the degree of mobility δ_m.
- $DDOF \leq \delta_M \leq DOF$
  - DOF governs the robot's ability to achieve various poses
  - DDOF governs its ability to achieve various paths
  - workspace vs. (robot/wheel) configuration space (c.f. Motion Planning)

Wheeled robots

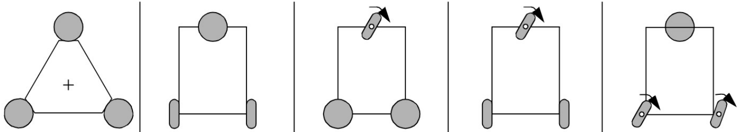
Holonomic vs. Nonholonomic

- Holonomic/Nonholonomic:
  - In mathematics, differential equations, functions and constraint expressions.
  - In mobile robotics, kinematic constraints of the robot chassis.
- Holonomic robot:
  - Has zero nonholonomic kinematic constraints.
  - Can directly achieve any state in their state space directly.
  - Examples: omni-steer, helicopter.
- Nonholonomic robot:
  - With one or more nonholonomic kinematic constraints.
  - Must use transition states to achieve any state in its state space.
  - Example: bicycle, car.

Wheeled robots

Holonomic vs. Nonholonomic

- An alternative way to describe a holonomic robot:
  - If and only if $DDOF = DOF$.



Omnidirectional	Differential	Omni-Steer	Tricycle	Two-Steer
$\delta_M = 3$	$\delta_M = 2$	$\delta_M = 3$	$\delta_M = 2$	$\delta_M = 3$
$\delta_m = 3$	$\delta_m = 2$	$\delta_m = 2$	$\delta_m = 1$	$\delta_m = 1$
$\delta_s = 0$	$\delta_s = 0$	$\delta_s = 1$	$\delta_s = 1$	$\delta_s = 2$

Wheeled robots

Beyond basic kinematics

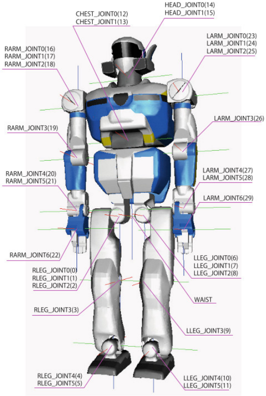
- For some robots, dynamic constraints must be expressed in addition to kinematic constraints.



Legged robots

Modeling

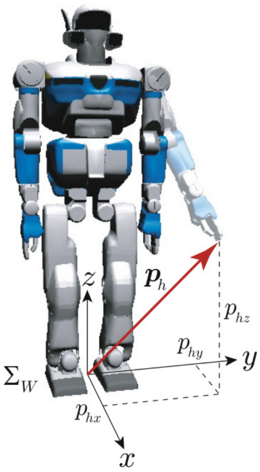
- Modeling similar to industrial robotic arms.
- Left: the Humanoid Robot HRP-2.
- Right: All joints have names and local coordinates.



Legged robots

Modeling: Local to world

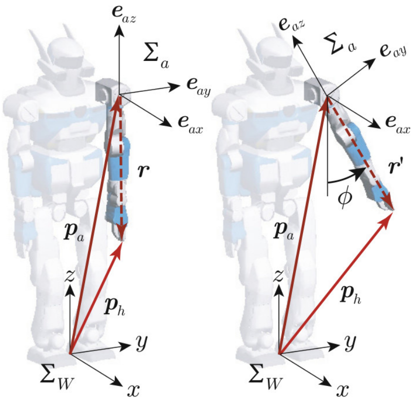
- World (or global) coordinate system (denoted Σ_W):
  - Origin: the intersection of the perpendicular line through the waist joint and the floor.
  - Fixed.
- Absolute position: a position defined in Σ_W.
  - For example, the tip of left hand: $p_h = \begin{bmatrix} p_{hx} \\ p_{hy} \\ p_{hz} \end{bmatrix}$



Legged robots

Modeling: Local to world

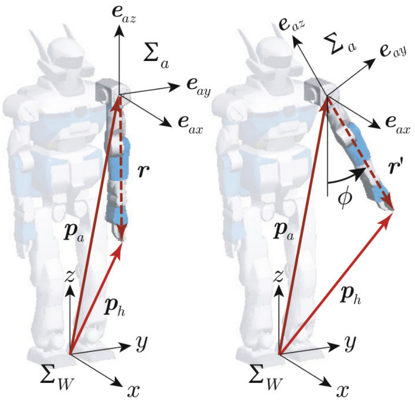
- Consider the hand tip position p_h changes due to the rotation of the shoulder:
  $p_h = p_a + r$
  - p_a: the absolute position of the left shoulder.
  - r: the hand tip position relative to the shoulder.
- When the arm is open:
  $p_h = p_a + r'$
  - The hand lifts by the vector's rotation from r to r'.
  - The shoulder keeps the same position p_a.



Legged robots

Modeling: Local to world

- Local coordinate system (denoted Σ_a):
  - Origin: left shoulder (in this case).
  - Movable.
- Left: initially, Σ_W and Σ_a are parallel.
- Right: Σ_a rotates together with the arm.

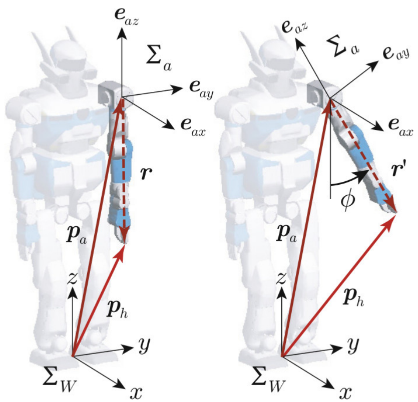


- $e_{ax} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_{ay} = \begin{bmatrix} 0 \\ \cos\phi \\ \sin\phi \end{bmatrix}$, $e_{az} = \begin{bmatrix} 0 \\ -\sin\phi \\ \cos\phi \end{bmatrix}$.

Legged robots

Modeling: Local to world

- Only e_{ay} and e_{az} are changed by ϕ: rotation around the x axis.
- Let's define a 3×3 matrix (i.e. rotation matrix): $R_a \equiv [e_{ax}, e_{ay}, e_{az}]$
- Then, a vector is rotated by: $r' = R_a \cdot r$

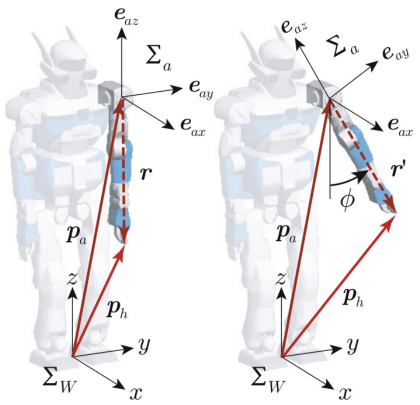


- $e_{ax} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_{ay} = \begin{bmatrix} 0 \\ \cos\phi \\ \sin\phi \end{bmatrix}$, $e_{az} = \begin{bmatrix} 0 \\ -\sin\phi \\ \cos\phi \end{bmatrix}$.

Legged robots

Modeling: Local to world

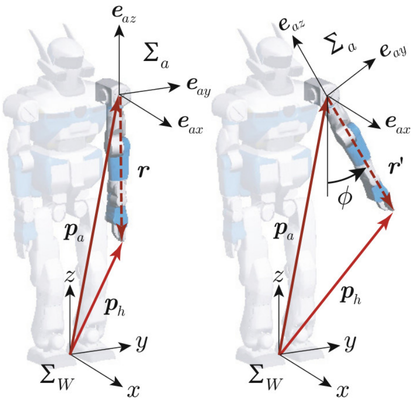
- Question: How to describe the relationship between the hand tip in the local coordinate system and the world coordinate system?
- Let's define the hand tip position in Σ_a as ${}^a p_h$, thus ${}^a p_h = r$
- Therefore, $p_h = p_a + R_a \cdot {}^a p_h$
- It can be rewritten as:
$$\begin{bmatrix} p_h \\ 1 \end{bmatrix} = \begin{bmatrix} R_a & p_a \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^a p_h \\ 1 \end{bmatrix}$$



Legged robots

Modeling: Local to world

- The 4×4 matrix (homogeneous transformation matrix) can be rewritten as:
$$T_a \equiv \begin{bmatrix} R_a & p_a \\ 0\ 0\ 0 & 1 \end{bmatrix}$$
- At the end: $\begin{bmatrix} p \\ 1 \end{bmatrix} = T_a \begin{bmatrix} {}^a p \\ 1 \end{bmatrix}$
- ${}^a p$ can be any point in the left arm.
- T_a describes the position and attitude of an object.



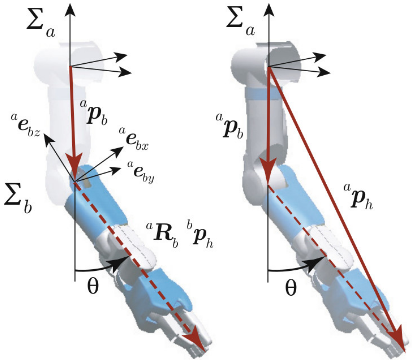
Legged robots

Modeling: Local to local

- Two local local coordinate systems: Σ_a (shoulder) and Σ_b (elbow).
- Suppose the elbow joint is rotated by θ degrees:

$$^a e_{bx} = \begin{bmatrix} \cos\theta \\ 0 \\ \sin\theta \end{bmatrix}, \; ^a e_{by} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$^a e_{bz} = \begin{bmatrix} -\sin\theta \\ 0 \\ \cos\theta \end{bmatrix}.$$

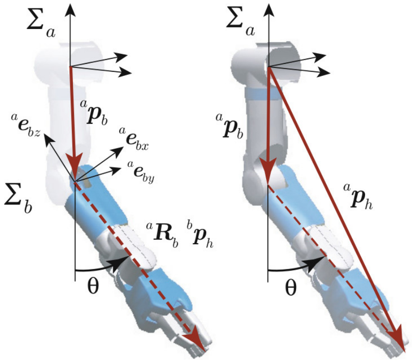


- The vectors are defined in σ_a.
- Only $^a e_{bx}$ and $^a e_{bz}$ are changed by θ: rotation around the y axis.

Legged robots

Modeling: Local to local

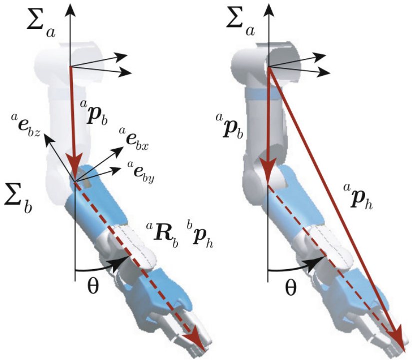
- Similar to "local to world":
  ${}^aR_b \equiv [{}^ae_{bx}, {}^ae_{by}, {}^ae_{bz}]$,
  $\begin{bmatrix} {}^ap_h \\ 1 \end{bmatrix} = {}^aT_b \begin{bmatrix} {}^bp_h \\ 1 \end{bmatrix}$, where
  ${}^aT_b \equiv \begin{bmatrix} {}^aR_b & {}^ap_b \\ 0\ 0\ 0 & 1 \end{bmatrix}$.
- Note that ${}^ap_b$ is the origin of Σ_b viewed from Σ_a.
- What about "from Σ_b to Σ_W"?



Legged robots

Modeling: Local to local

- $\begin{bmatrix} p_h \\ 1 \end{bmatrix} = T_a {}^a T_b \begin{bmatrix} {}^b p_h \\ 1 \end{bmatrix}$
- $T_b \equiv T_a {}^a T_b$
  - T_b: homogeneous transformation matrix for "Σ_b to Σ_W".
  - T_a: amount of shoulder rotation.
  - ${}^a T_b$ varies with the amount of elbow rotation.



- Chain rules (multiply from the right):
  $T_N = T_1\, {}^1T_2\, {}^2T_3\, \ldots\, {}^{N-1}T_N$

Summary

- Basic kinematics of mobile robots.
- Current general trend of mobile manipulators composed of mobile robot bases and robotic arms.
- Further reading:
  - Legged robots: absolute attitude, absolute velocity.
  - Wheeled robots: wheel constrains, kinematic control.

The end

Thank you for your attention!

Any questions?